$$E = mc^2$$

$$= \hbar\omega$$

Holger Müller

Kioloa 2011

# Compton oscillators + time dilation = de Broglie (1924) relations


$$h\nu_0 = m_0 c^2$$

$\nu_0$ being measured, of course, in a system which is at rest with respect to a certain amount of energy. This hypothesis is the base of our system, and it is

Then the frequency observed by the fixed observer will be

$$\nu_1 = \nu_0 (1-\beta^2)^{1/2} = \frac{m_0 c^2}{h}(1-\beta^2)^{1/2}$$

On the other hand, since the energy of the moving body is equal to $m_0c^2/(1-\beta^2)^{1/2}$ for the same observer, the corresponding frequency according to the relationship of the quantum is

$$\nu = h^{-1}[m_0c^2/(1-\beta^2)^{1/2}].$$

The two frequencies $\nu_1$ and $\nu$ are essentially different since the factor $(1-\beta^2)^{1/2}$ is not involved in the same way. This is a difficulty that has intrigued me for a long time; I have succeeded in eliminating it by demonstrating the following theorem that I shall call the theorem of the

Translated in Am. J. Phys. **40,** 1315 (1972)

# De Broglie + Gravity

Particle represents an oscillator

$$\omega_C = \frac{mc^2}{\hbar}.$$

Time dilation

$$\omega_C \to \omega_C \sqrt{1 - v^2 / c^2}$$

Gravitational redshift

$$\omega_C \to \omega_C \left(1 - \frac{\Delta U}{c^2}\right)$$

Time dilation + redshift

$$\omega_C \to \omega_C \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$

# Proper time ~ action

Proper time $$c\,d\tau = \sqrt{-g_{\alpha\beta}\,dx^{\alpha}dx^{\beta}}$$

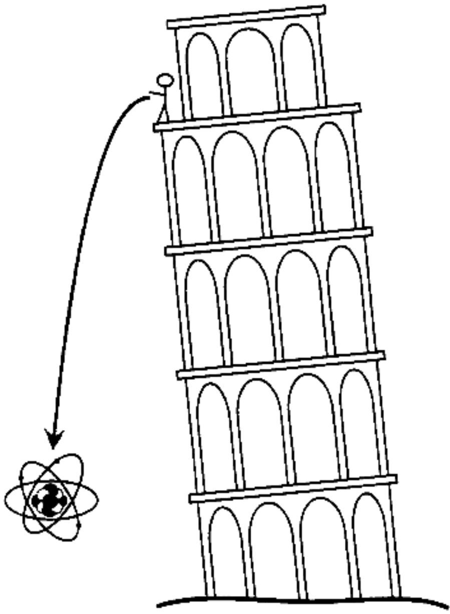
Action $$S = mc^2 \int d\tau$$

Path integral $$\Psi(t^B;\vec{x}^B) = \int Dq \exp\left(\frac{i}{\hbar}S\right)\Psi(t^A;\vec{x}^A)$$

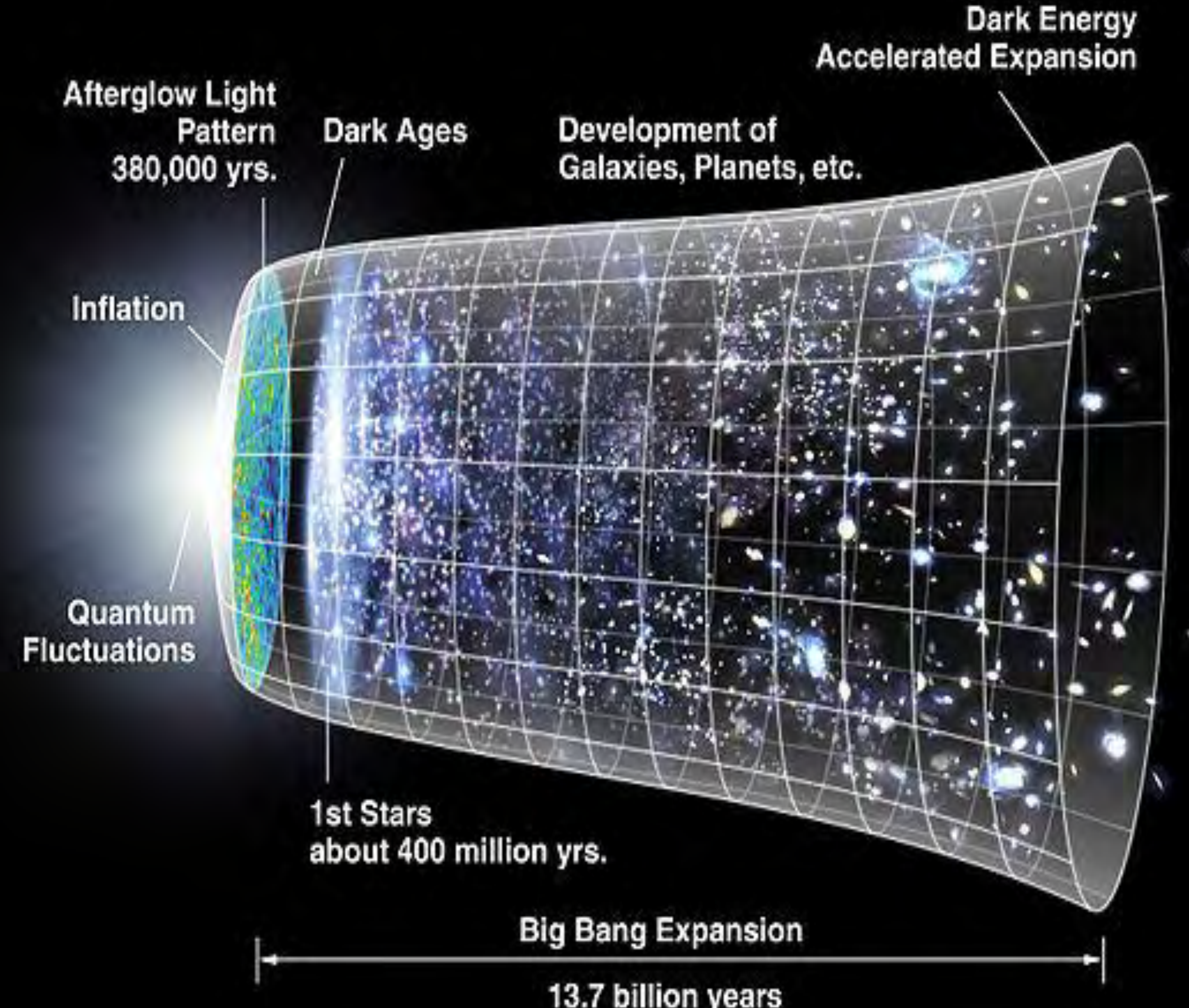
Compton oscillators + general relativity = quantum mechanics

# Outline

1. Motivation
2. Atom interferometer redshift test
3. Really? Don't you just measure *g*, not *U*?
4. Aharonov-Bohm effect
   - we measure *U*
5. Compton clock
   - particles are *really* clocks
6. What is time?

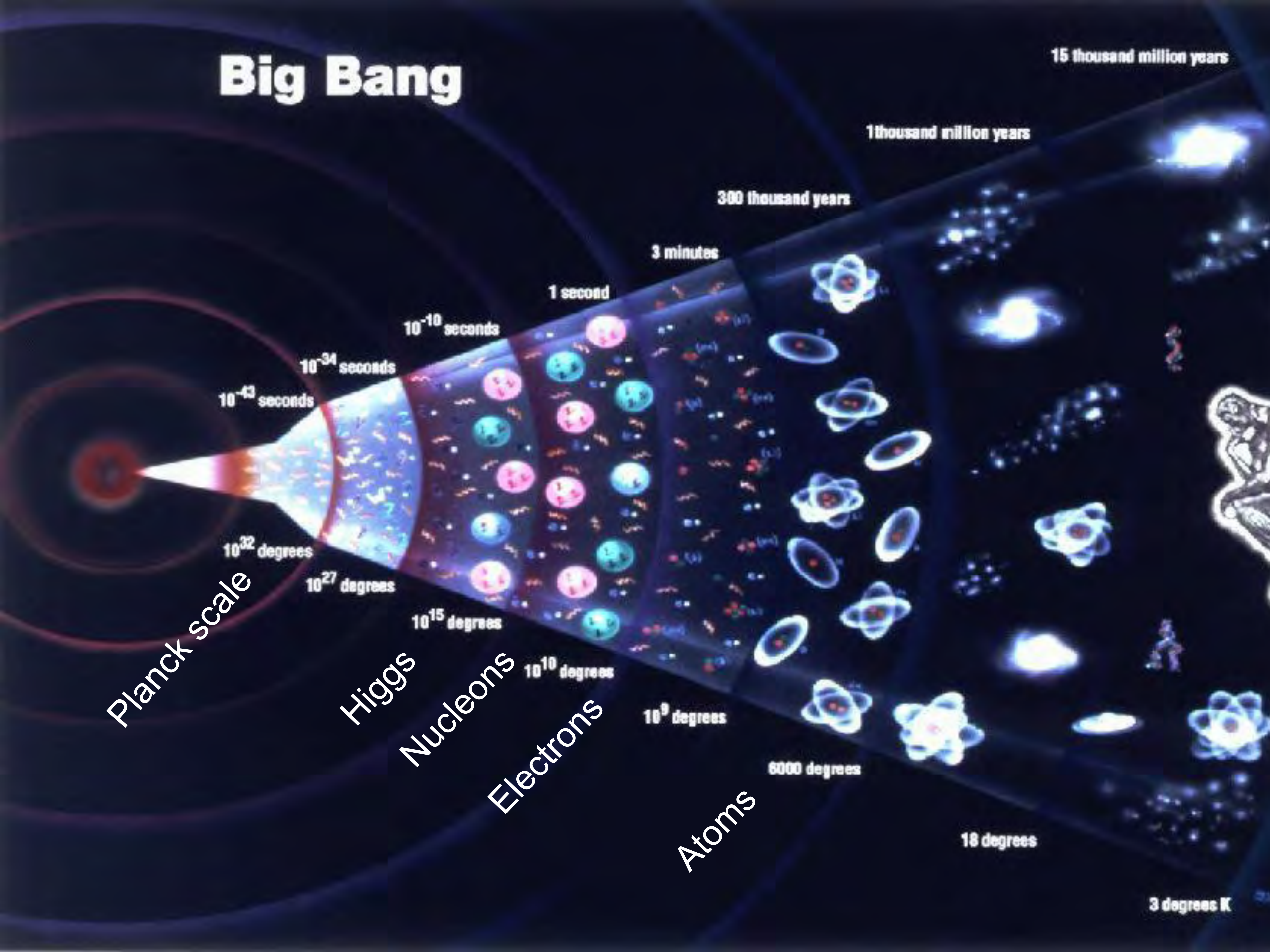


A. Peters



Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Wikipedia



Big Bang
15 thousand million years
1thousand million years
300 thousand years
3 minutes
1 second
$10^{-10}$ seconds
$10^{-34}$ seconds
$10^{-43}$ seconds
$10^{32}$ degrees
$10^{27}$ degrees
$10^{15}$ degrees
$10^{10}$ degrees
$10^{9}$ degrees
6000 degrees
18 degrees
3 degrees K
Planck scale
Higgs
Nucleons
Electrons
Atoms

# Penrose's argument

If you have nothing that has rest mass, you can't have a clock. And you can't measure distances either.

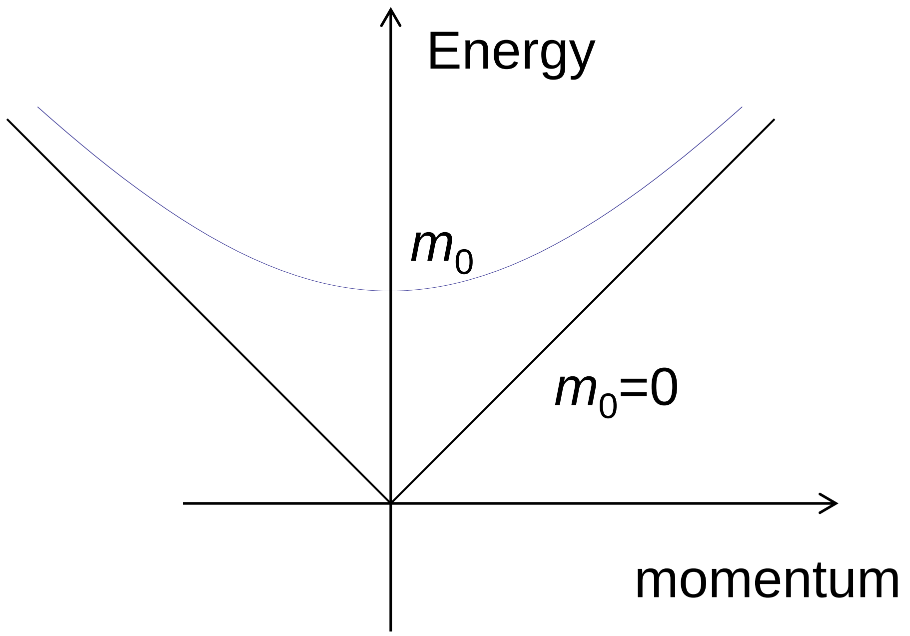
Maxwell equations are scale invariant

- Beginning: All *m=0.*

- Big freeze: only photons left.

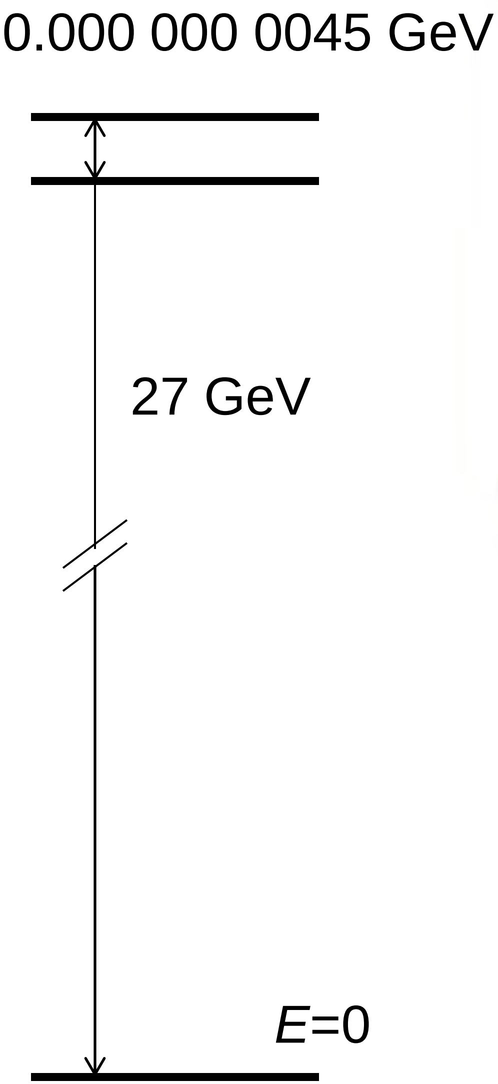
Again, $m$=0

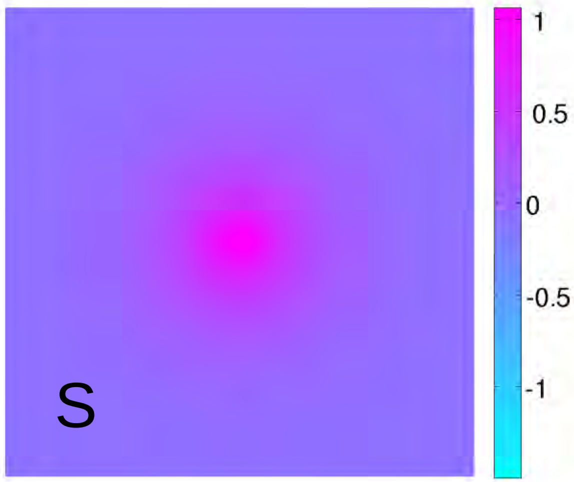
"The scale changed," but there is nothing to measure scales



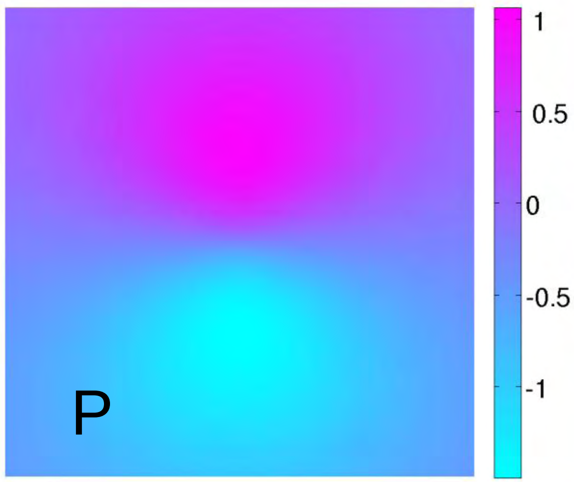

Is a rock a clock?

# "Ticking" of an atomic ($^{27}Al^+$) clock

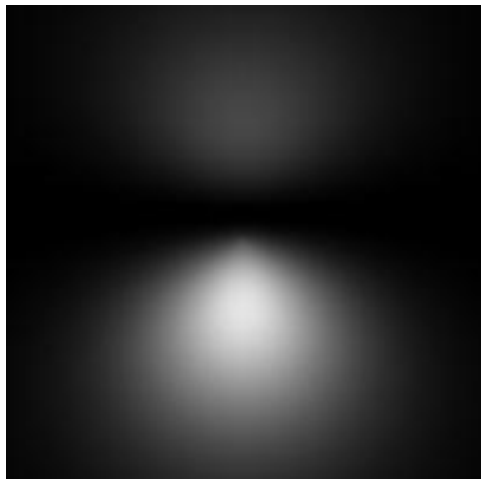



$+e^{-i\omega t}$




=



$\omega = 2\pi \times 1.121 \times 10^{15}$ Hz

Thanks to Till Rosenband!

# $E = mc^2 = h\nu$ provides a clock


Action $$S = \int mc^2 d\tau,$$

Propagator $$\langle \psi(t_1) | \psi(t_2) \rangle = \int Dq \exp(iS/\hbar).$$

Quantum phase $$\varphi = \frac{i}{\hbar} \int mc^2 d\tau = \int \omega_C d\tau,$$

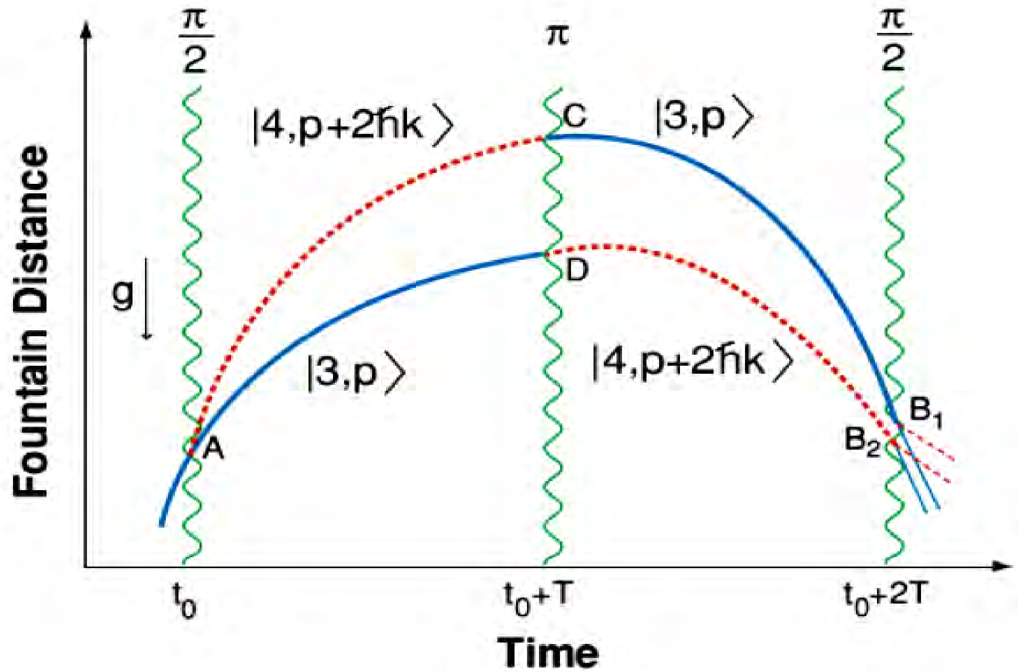
$$d\tau / dt = \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}.$$

Quantum phase proportional to proper time (!)

[Misner, Thorne & Wheeler (1970); Feynman & Hibbs, (1965); Borde, *Eur. Phys. J. Special Topics* (2008).]

# Atom interferometer redshift test

# Atom Interferometer




$$\Delta\varphi = \frac{mc^2}{\hbar}\int\left(\frac{\varphi_s - \varphi_e}{c^2} - \frac{\vec{v}_s^{\,2} - \vec{v}_e^{\,2}}{2c^2}\right)dt + \sum \varphi_{laser}$$

Redshift
$kgT^2$

Time dilation
$-kgT^2$

Laser interaction
$kgT^2$

related by $v = dx/dt$

# History

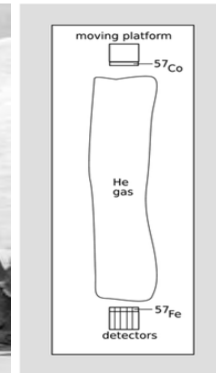
Einstein[1] 1911: solar spectral lines. Didn't work.

*Einfluß der Schwerkraft auf die Ausbreitung des Lichtes.* 905

müssen also die Spektrallinien des Sonnenlichtes gegenüber den entsprechenden Spektrallinien irdischer Lichtquellen etwas nach dem Rot verschoben sein, und zwar um den relativen Betrag

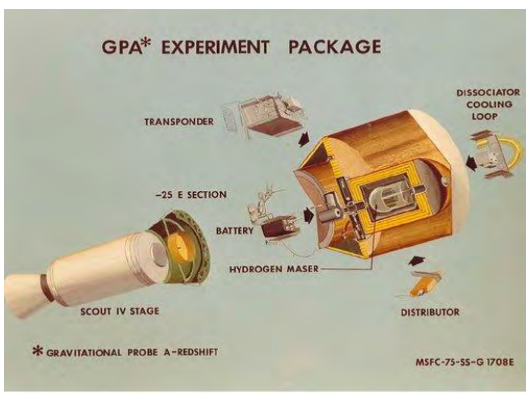
$$\frac{\nu_0 - \nu}{\nu_0} = \frac{-\Phi}{c^2} = 2\cdot 10^{-6}.$$

Wenn die Bedingungen, unter welchen die Sonnenlinien entstehen, genau bekannt wären, wäre diese Verschiebung noch der Messung zugänglich. Da aber anderweitige Einflüsse (Druck, Temperatur) die Lage des Schwerpunktes der Spektrallinien beeinflussen, ist es schwer zu konstatieren, ob der hier




Pound & Rebka,[2] 1960: Mossbauer effect. $10^{-2}$.

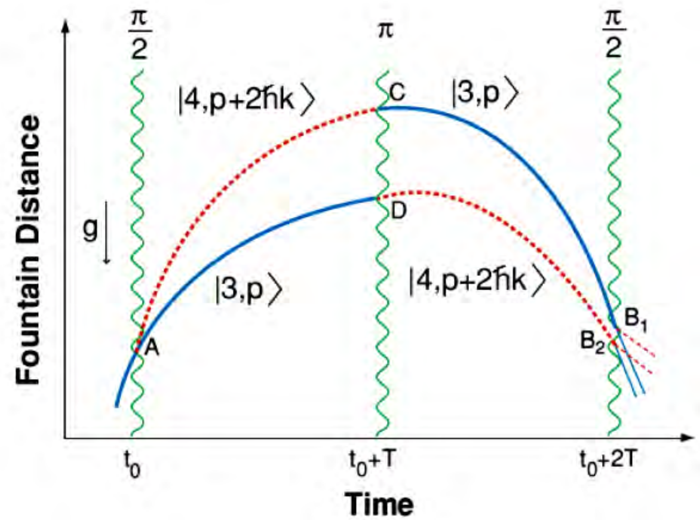
Vessot *et al.*,[4] 1976: Atomic clocks in rocket $10^{-5}$.




Hafele & Keating,[3] 1971: Atomic clocks in aircraft $10^{-1}$.

[1] Ann. Phys. **340,** 898 (1911). [2] PRL **4,** 337 (1960); **13,** 539 (1964); Phys. Rev. **140,** B788 (1965).
[3] Science **177,** 166 (1972): 177, 168 (1972). [4] PRL **45,** 2081 (1980).

# Atom Interferometer




Redshift  Time dilation  Laser

$$\Delta\varphi = \frac{mc^2}{\hbar}\int\left(\frac{\varphi_s - \varphi_e}{c^2} - \frac{\vec{v}_s^{\,2} - \vec{v}_e^{\,2}}{2c^2}\right)dt + \sum_{i=1}^{3} k_i z(t_i)$$

=0
(non-gravitational
energy conservation)

Alternative Interpretation

- If *any* energy is conserved, first terms also cancel
- then, testing UFF = testing redshift
- AI becomes indirect measurement of g
- As any redshift measurement

nature

Vol 463|18 February 2010|**doi:10.1038/nature08776**

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# A precision measurement of the gravitational redshift by the interference of matter waves

Holger Müller[1,2], Achim Peters[3] & Steven Chu[1,2,4]

**One of the central predictions of metric theories of gravity, such as general relativity, is that a clock in a gravitational potential *U* will run more slowly by a factor of $1 + U/c^2$, where *c* is the velocity of light, as compared to a similar clock outside the potential[1]. This effect, known as gravitational redshift, is important to the operation of the global positioning system[2], timekeeping[3,4] and future experiments with ultra-precise, space-based clocks[5] (such as searches for variations in fundamental constants). The gravitational redshift has been measured using clocks on a tower[6], an aircraft[7] and a rocket[8], currently reaching an accuracy of $7 \times 10^{-5}$. Here we show that laboratory experiments based on quantum interference of atoms[9,10] enable a much more precise measurement, yielding an accuracy of $7 \times 10^{-9}$. Our result supports the view that gravity is a manifestation of space-time curvature, an underlying principle of general relativity that has come under scrutiny in connection with the search for a theory of quantum gravity[11]. Improving the redshift measurement is particularly important because this test has been the least accurate among the experiments that are required to support curved space-time theories[1].**

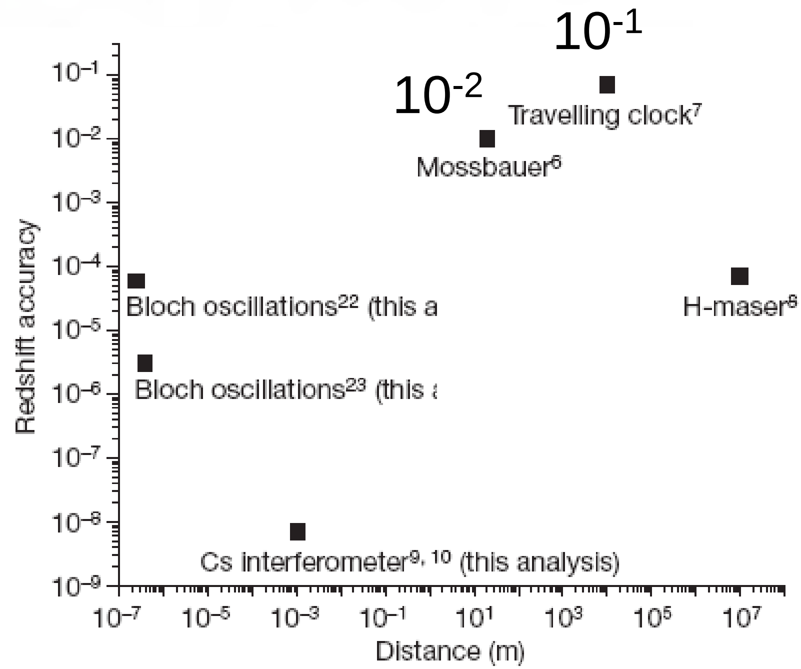


**Figure 2 | Absolute determinations of the gravitational redshift.** The accuracy (defined as the standard error) in $\beta$ is plotted versus the relative height of the clocks.

# Criticism

- $\omega_C$ is unphysical: the AI is not a pair of clocks, but just a measurement of *g*

- The atom is no real clock: it cannot be used to lock an oscillator.

**Does an atom interferometer test the gravitational redshift at the Compton frequency?**

Peter Wolf,[1] Luc Blanchet,[2] Christian J. Bordé,[1,3] Serge Reynaud,[4] Christophe Salomon,[5] and Claude Cohen-Tannoudji[5]

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[2]*GRECO, Institut d'Astrophysique de Paris, CNRS UMR 7095, UPMC, 98bis boulevard Arago, 75014 Paris, France*
[3]*Laboratoire de Physique des Lasers, CNRS UMR 7538, 99 avenue Jean-Baptiste Clément, 93430 Villetaneuse, France*
[4]*Laboratoire Kastler Brossel, CNRS UMR 8552, ENS, UPMC, Campus Jussieu, 75252 Paris, France*
[5]*Laboratoire Kastler Brossel et Collège de France, CNRS UMR 8552, ENS, UPMC, 24 rue Lhomond, 75231 Paris, France*

(Dated: May 13, 2011)

Abstract

Atom interferometers allow the measurement of the acceleration of freely falling atoms with respect to an experimental platform at rest on Earth's surface. Such experiments have been used

# Equivalence Principle and Gravitational Redshift

Michael A. Hohensee,[1,*] Steven Chu,[1,†] Achim Peters,[2] and Holger Müller[1]
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(Received 17 February 2011; published 11 April 2011)

We investigate leading order deviations from general relativity that violate the Einstein equivalence principle in the gravitational standard model extension. We show that redshift experiments based on matter waves and clock comparisons are equivalent to one another. Consideration of torsion balance tests, along with matter-wave, microwave, optical, and Mössbauer clock tests, yields comprehensive limits on spin-independent Einstein equivalence principle-violating standard model extension terms at the $10^{-6}$ level.

DOI: 10.1103/PhysRevLett.106.151102 PACS numbers: 04.80.−y, 03.30.+p, 11.30.Cp, 12.60.−i

TABLE I. Sensitivity of redshift experiments. The EEP-violation signal for each experiment is given as a linear combination of SME parameters. The observable for the Pound-Rebka Mössbauer test, e.g., is $-1.1\ \mathrm{GeV}^{-1}\alpha(\bar{a}^n_{\mathrm{eff}})_0 - 1.1\ \mathrm{GeV}^{-1}\alpha(\bar{a}^{e+p}_{\mathrm{eff}})_0 + (-0.34 + [-0.66])(\bar{c}^n)_{00} + (-0.34 + [-0.006])(\bar{c}^p)_{00} + 0.0002(\bar{c}^e)_{00}$, with $\bar{a}^{e+p}_{\mathrm{eff}} = \bar{a}^p_{\mathrm{eff}} + \bar{a}^e_{\mathrm{eff}}$. The last column shows the measured value and $1\sigma$ uncertainty. Signals dependent on models for $\xi$ are in square brackets. Curly brackets mark expected limits.

| Method | $\alpha(\bar{a}^n_{\mathrm{eff}})_0$ GeV | $\alpha(\bar{a}^{e+p}_{\mathrm{eff}})_0$ GeV | $(\bar{c}^n)_{00}$ | $(\bar{c}^p)_{00}$ | $(\bar{c}^e)_{00}$ | Limit ppm |
|---|---|---|---|---|---|---|
| Mössbauer effect [2] | −1.072 | −1.072 | 0.3358 − [2/3] | −0.3353 − [0.006] | 0.000 182 6 | 1000 ± 7600 |
| H maser on rocket [3] | −1.072 | −1.072 | 0.3358 | 0.3353 − [0.67] | 0.000 182 6 − [1.3] | 2.5 ± 70 |
| Cs fountain (*proj.*) [16] | −1.072 | −1.072 | 0.3358 + [0.40] | 0.34 + [0.28] | 0.000 182 6 − [1.3] | {2} |
| Bloch oscillations [4,17] | 0.1632 | −0.1580 | −0.051 12 − [0.0005] | 0.049 40 + [0.0010] | 0.000 026 90 | 3 ± 1 |
| Bloch oscillations [6] | 0.1492 | −0.1439 | −0.046 73 − [0.0006] | 0.045 00 + [0.0008] | 0.000 024 51 | 0.16 ± 0.14 |
| Cs interferometer [4] | 0.1881 | −0.1835 | −0.058 90 − [0.0004] | 0.057 39 + [0.001] | 0.000 031 26 | 0.007 ± 0.007 |
| Rb interferometer [18] | 0.1632 | −0.1580 | −0.051 12 − [0.0005] | 0.049 40 + [0.001] | 0.000 026 90 | −0.004 ± 0.007 |

# Bottom line

- Comprehensive limits on all 5 coefficients giving EEP-violations (ppm)

| | $(a^n_{eff})_0$ | $(a^p_{eff})_0 + (a^n_{eff})_0$ | $(c^n_{eff})_{00}$ | $(c^p_{eff})_{00}$ | $(c^e_{eff})_{00}$ |
|---|---|---|---|---|---|
| | GeV | GeV | | | |
| AI+clocks | 4.3±3.7 | 0.8±1.0 | 7.6±6.7 | -3.3±3.5 | 4.6±4.6 |
| | | | | | |

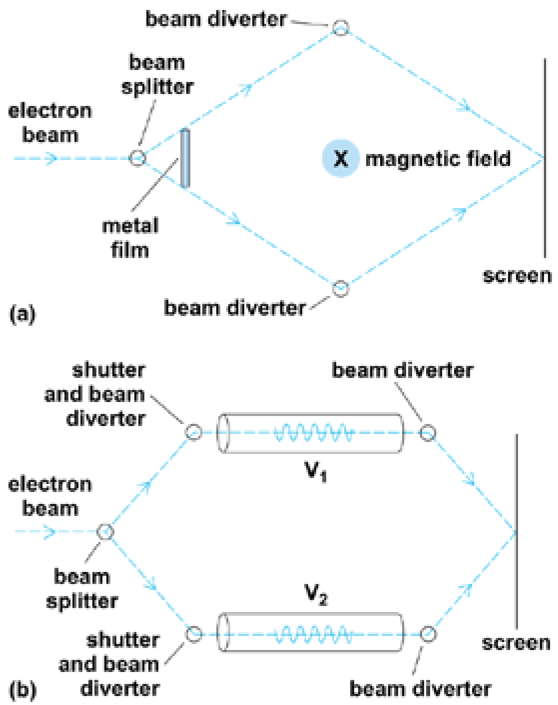
- A sixth coeffcient, $(a^p_{eff})_0 + (a^n_{eff})_0$, is only relevant for charged particles

- Neglected anomalies coupling to pions and mass dimension 5 operators.

- Previously, only one limit known: $|0.8(a^n_{eff})_0 + (a^p_{eff})_0 + (a^n_{eff})_0| < 1\text{x}10^{-11}$ GeV

# Gravitational Aharonov-Bohm Effect

# Aharonov-Bohm effect

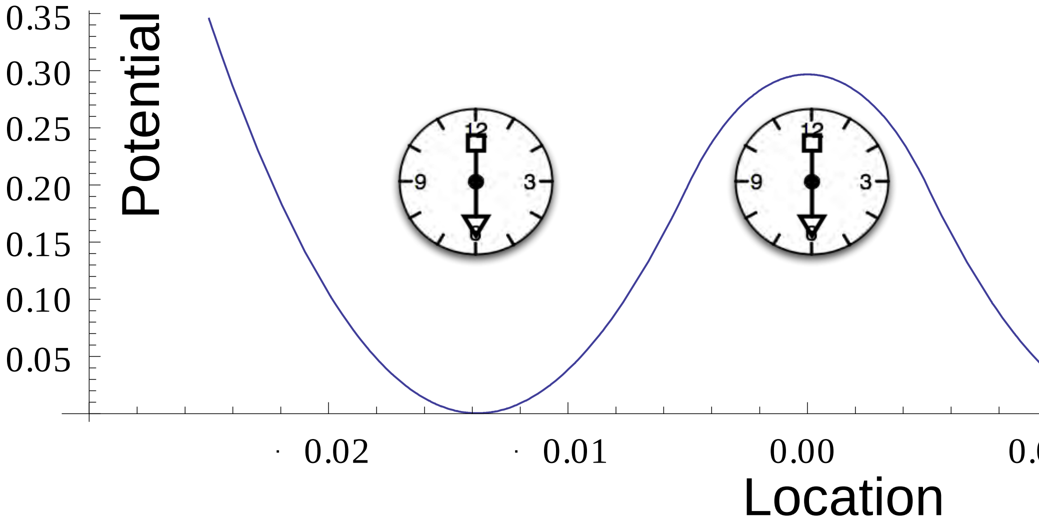


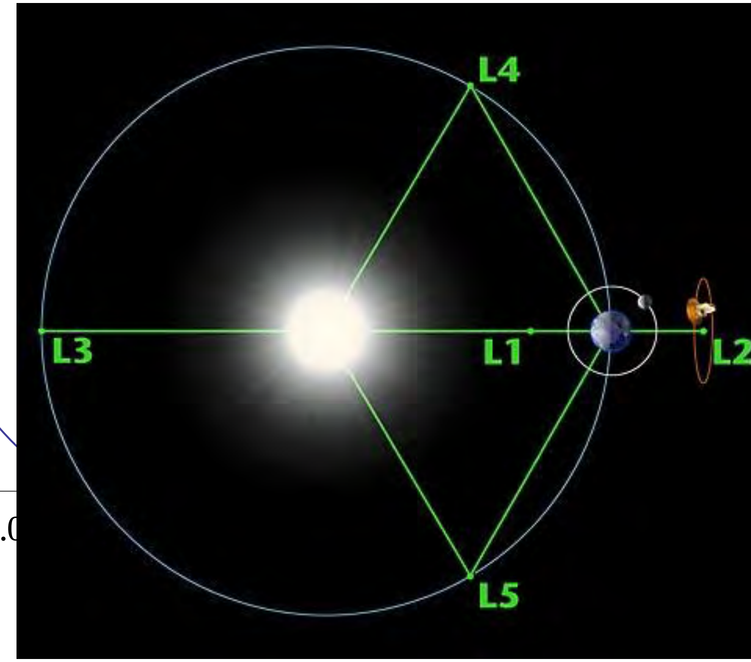

$$\varphi_A = -\frac{e}{\hbar}\int \vec{A}\cdot d\vec{l}$$

$$\varphi_V = \frac{e}{\hbar}\int V dt,$$

- No classical force
- Non-dispersive: not related to displacement or distortion of the wave packet
- **Topological:** cannot be predicted from any number of local measurements anywhere on the path

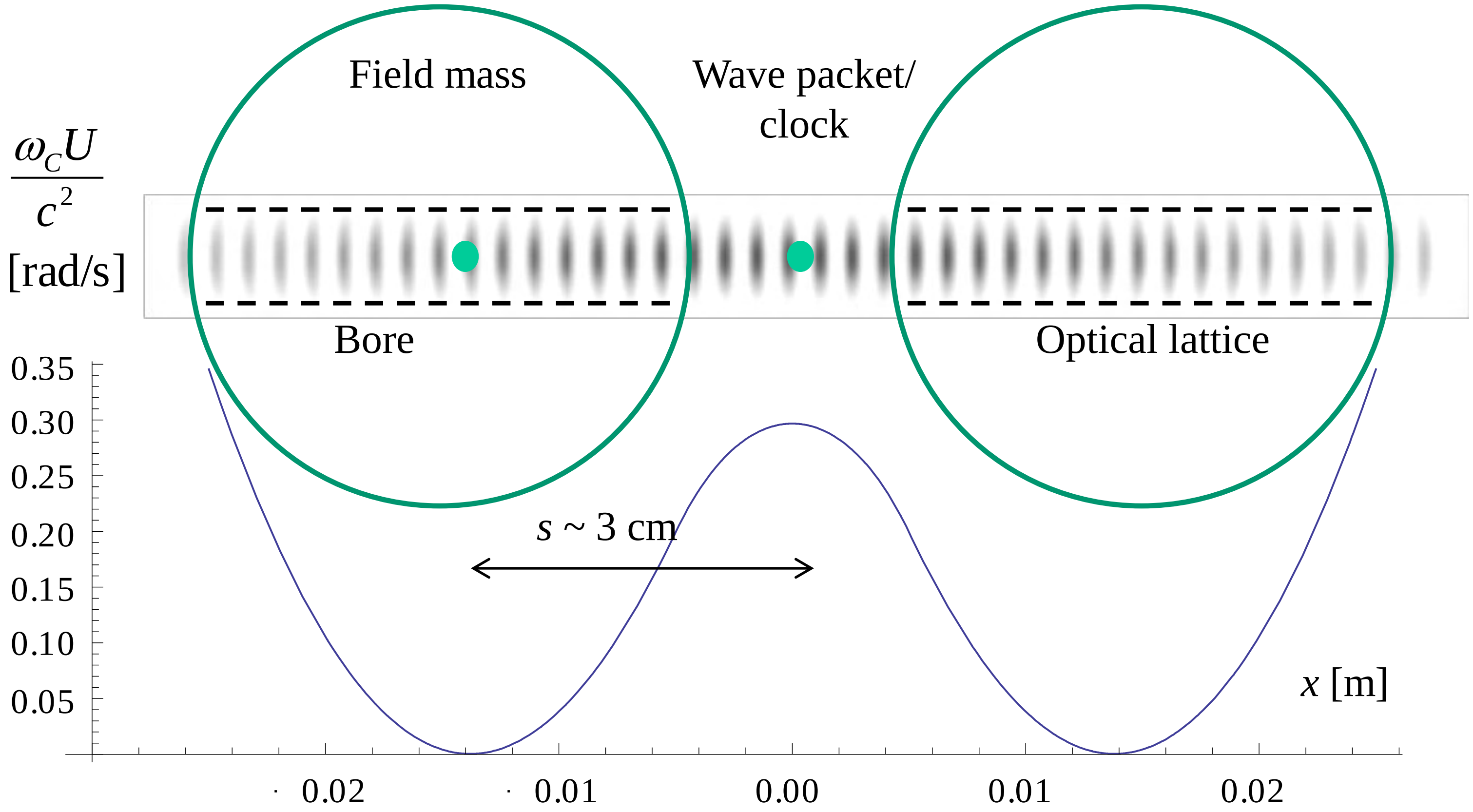
# Gravity's Aharonov-Bohm effect



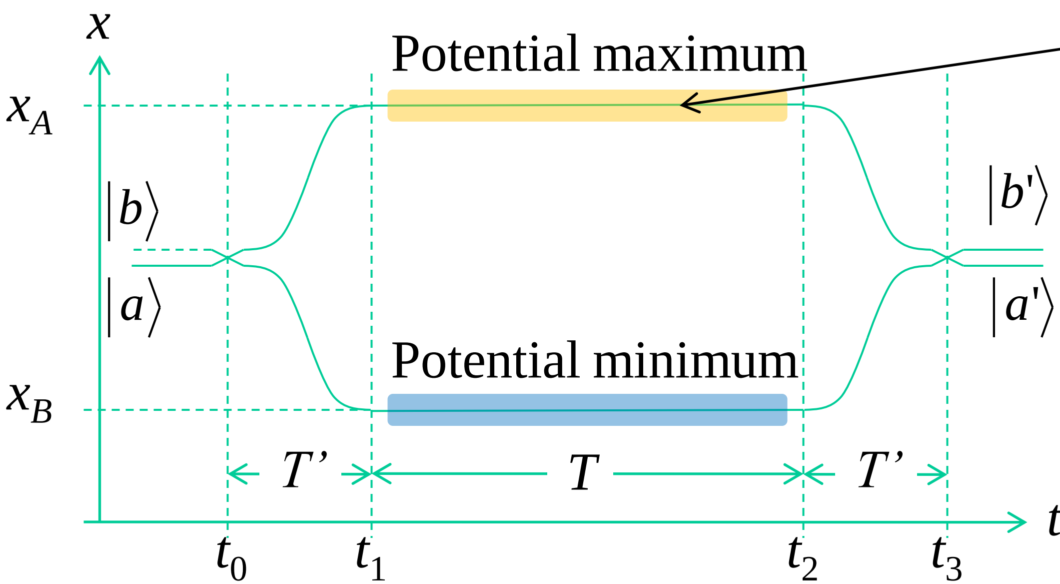

- No forces at saddle points
- Still causes redshift => phase shift between clocks
- Terrestrial experiment: $\varrho$=10 g/cm$^3$, R=10 m: $\Delta\nu/\nu = 5\text{x}10^{-21}$
- Possible realization: Earth-moon Lagrange points

# Realization with atom-clocks




$$\varphi = \omega_C \int \frac{\Delta U}{c^2} dt = 0.16 \left(\frac{s}{\mathrm{cm}}\right)^2 \left(\frac{\rho}{10\ \mathrm{g/cm}^3}\right) \left(\frac{m}{m_{Cs}}\right) \left(\frac{T}{\mathrm{s}}\right)$$

# Properties




No gravimeters on this path will register the potential

Gauge-dependent integrand

$$\Delta\varphi = \omega_C \int \frac{U}{c^2}\, dt$$

- Topological
- Non-dispersive
- Phase shift is a consequence of the potential, not the force
- "Shaking" the path can prove time dilation as well.

- Both relativistic effects on time apply to atom-clocks
- AI is not a measurement of g

# Derivation

"Quantum" picture

Time evolution

$$|\psi_{A,B}(t_2)\rangle = e^{\frac{i}{\hbar}H_{A,B}T}|\psi_{A,B}(t_1)\rangle.$$

(no field-generating masses)

$$|\psi_{A,B}(t_2)\rangle = e^{\frac{i}{\hbar}[H_{A,B}T+mU(x_{A,B})T]}|\psi_{A,B}(t_1)\rangle.$$

(with field masses)

$$\phi_G = m\Delta UT/\hbar.$$

(differential signal)

General relativity picture

Proper time

$$\Delta\tau_0 = \frac{1}{c^2}\int [U_0(x_A) - U_0(x_B)]dt,$$

$$\Delta\phi_G = \omega\Delta\tau_G = \omega\Delta UT/c^2,$$

(differential signal)

The phase shifts are identical for clocks of $\omega_C = mc^2/\hbar$

# Systematic effects

- Residual force: displacement, not acceleration $\delta x = F/(2k^2V_0)$

Causes potential change, thus phase shift $F^2T/(4k^2V_0\hbar)$

E.g., Earth's gravity $8gGm_{\mathrm{Cs}}^2\pi R^3\rho T/(3\hbar k^2L^2V_0) \simeq 2 \times 10^{-9}(T/\mathrm{s})$

- Curvature of potential

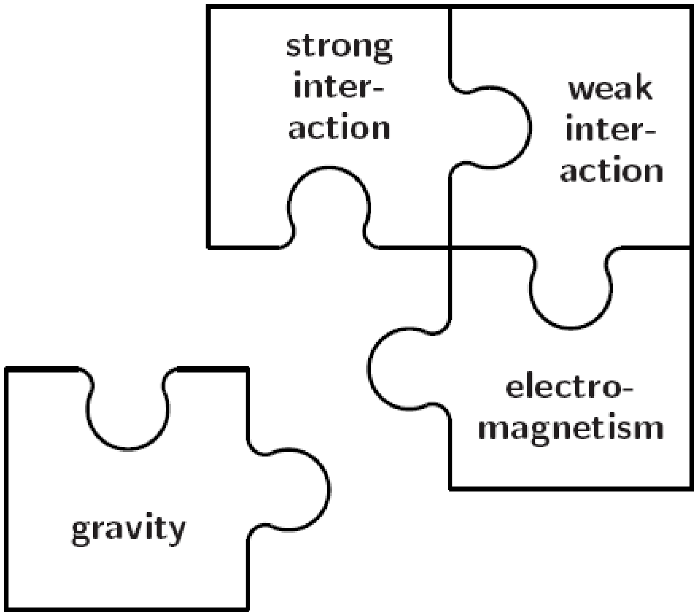
Lattice

Atoms in harmonic oscillator states $\omega_i^2 = (\partial^2V/\partial x_i^2)/(2m)$

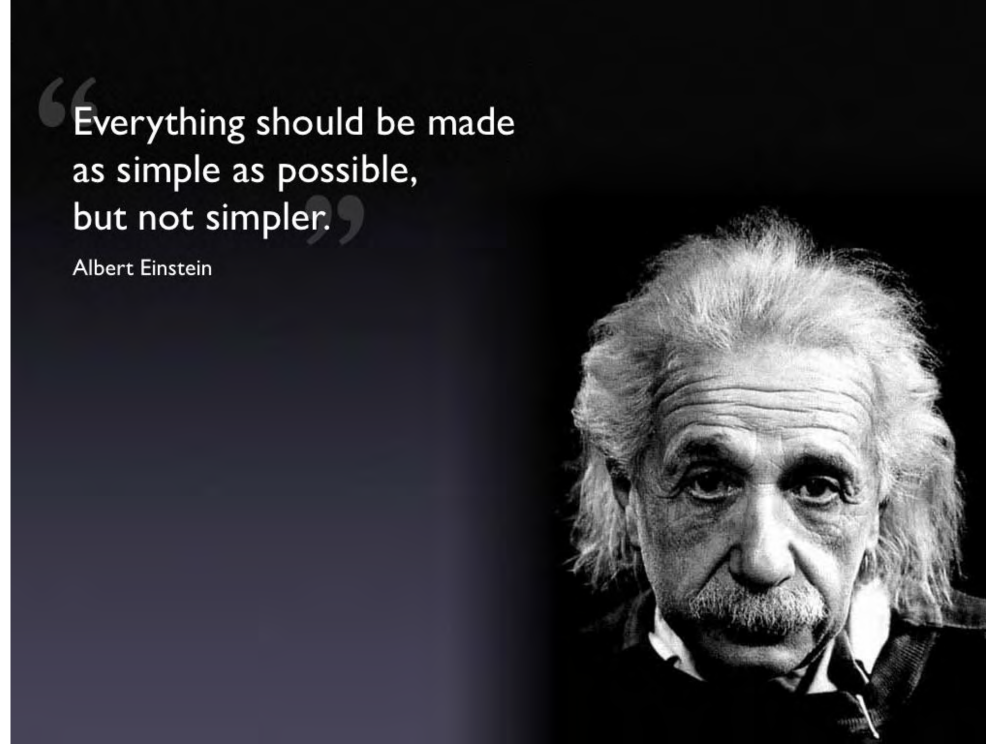
Curvature of gravitational potential: V -> U + V $\Delta E_h = \sum_i \frac{\hbar}{2\omega_i}\partial^2U/(\partial x_i)^2$

Changes

$(2/3)\pi G\rho/\omega_i \sim 2\times 10^{-6}\,\mathrm{rad/s},$




C. Lammerzahl




General Relativity:

• Theory of gravity, space & time used in GPS, astrophysics, cosmology, string theory,…

• Nonlinear theory: gravity causes more gravity

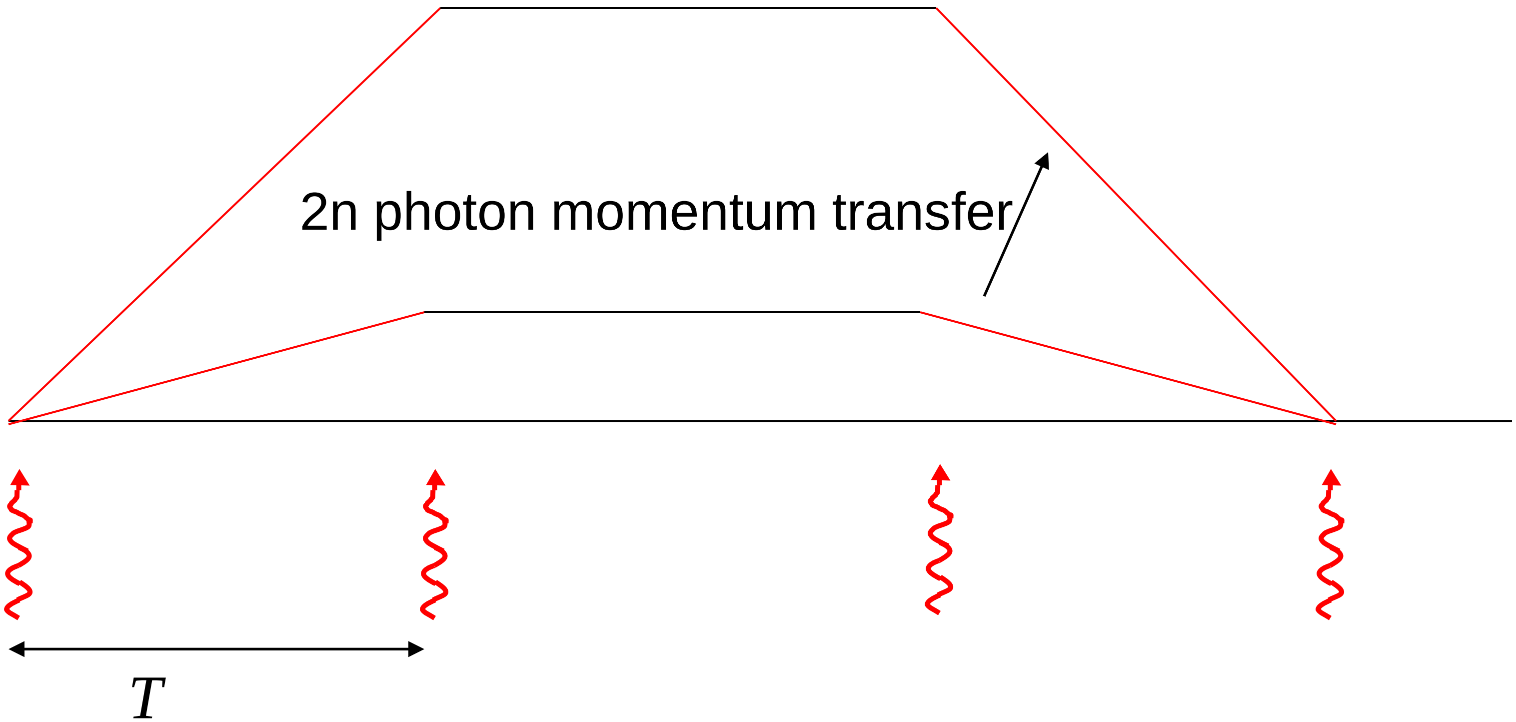
• Cannot be quantized

• Low-energy limit of string theory might be Lorentz-violating [V.A. Kostelecky …]

# Compton clock

$$\omega = \frac{mc^2}{\hbar}\frac{1}{n^2}$$

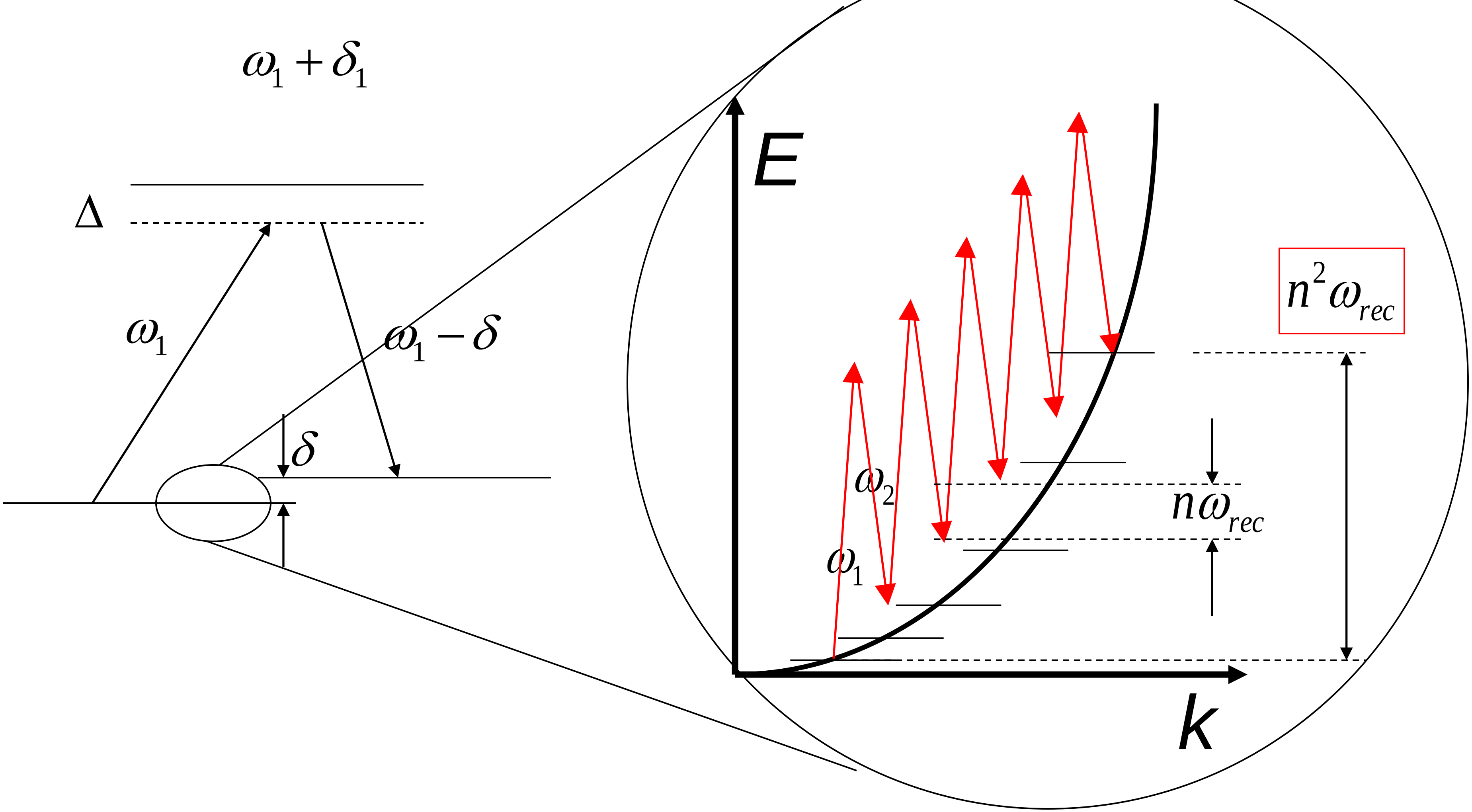
# Ramsey-Borde interferometer



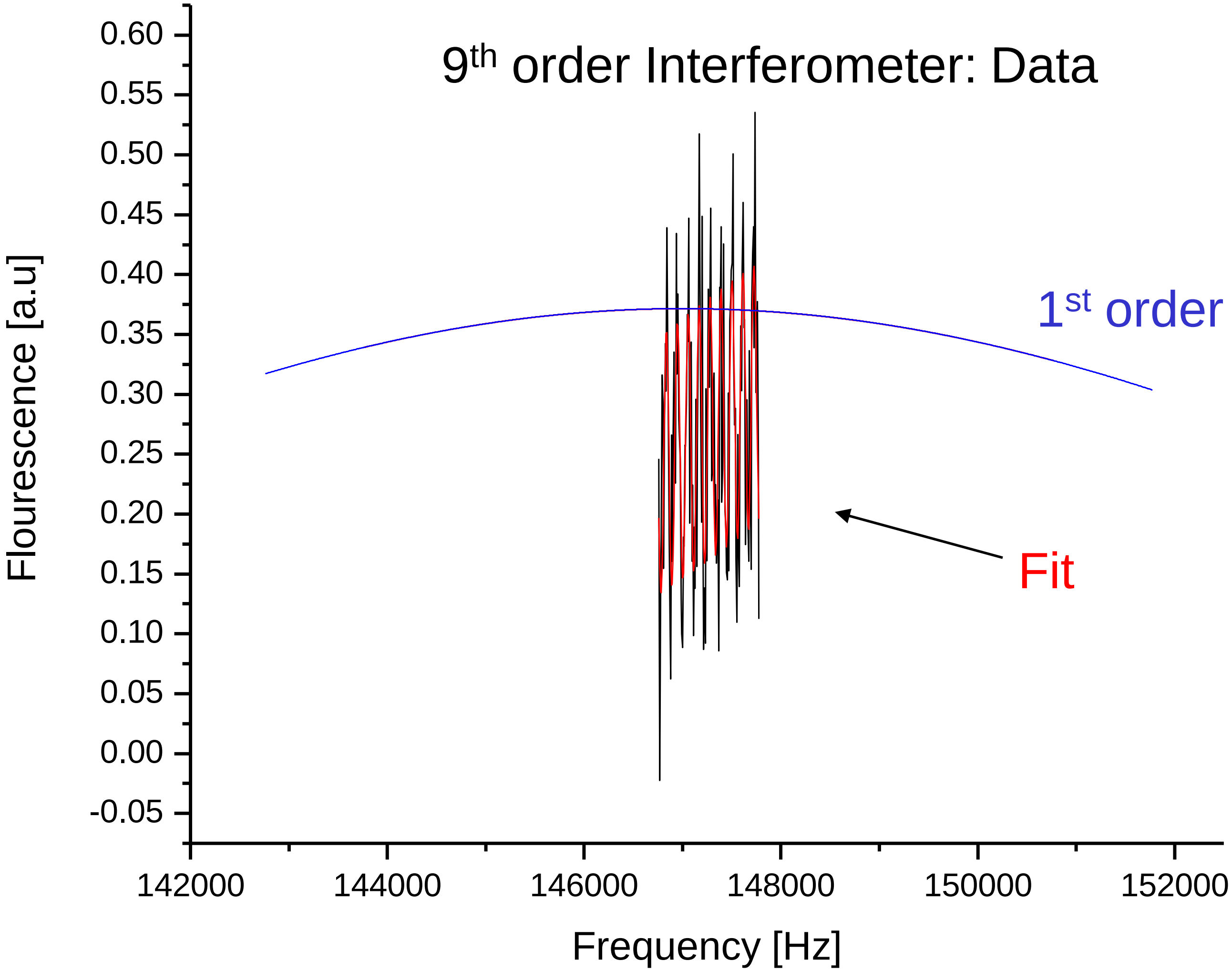

$$\Phi_1 - \Phi_2 = 16 n^2 \omega_r T$$

$$\omega_r = \frac{\hbar k^2}{2m}$$

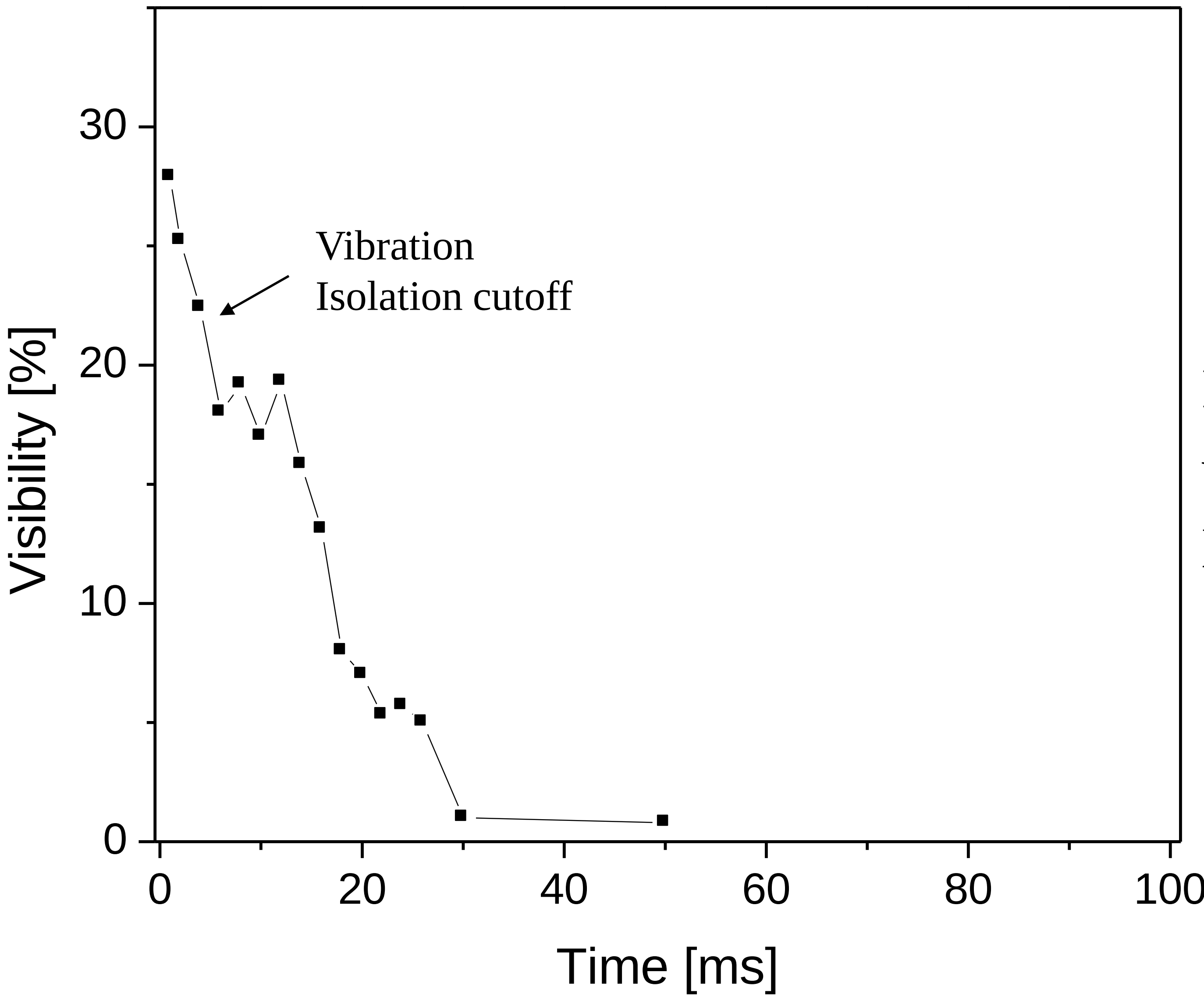
# Multiphoton Bragg diffraction



# 18 photon vs. 2 photon RB Interferometer

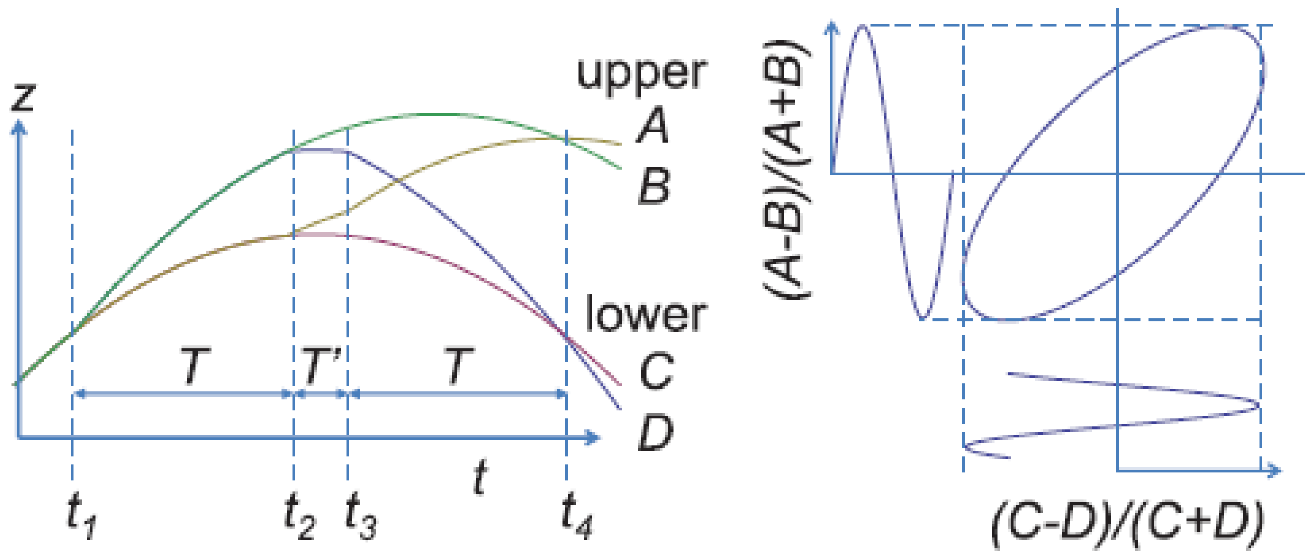


# Problem: Contrast decay




Limitation:
vibrational
phase noise

# Solution: Simultaneous conjugate Interferometers

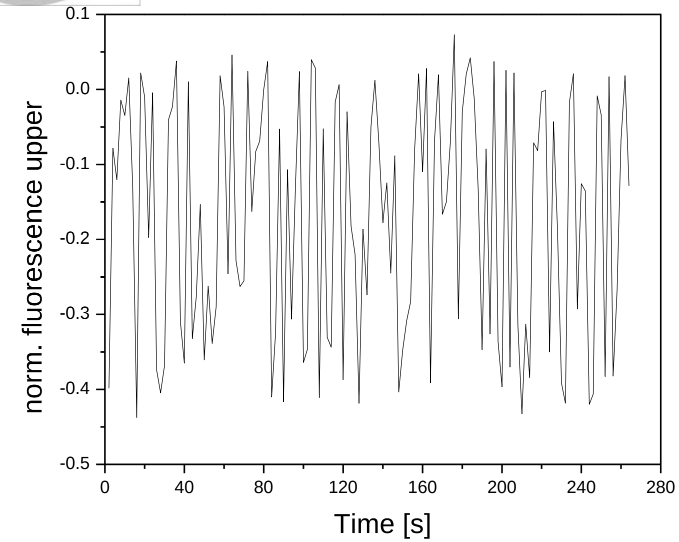


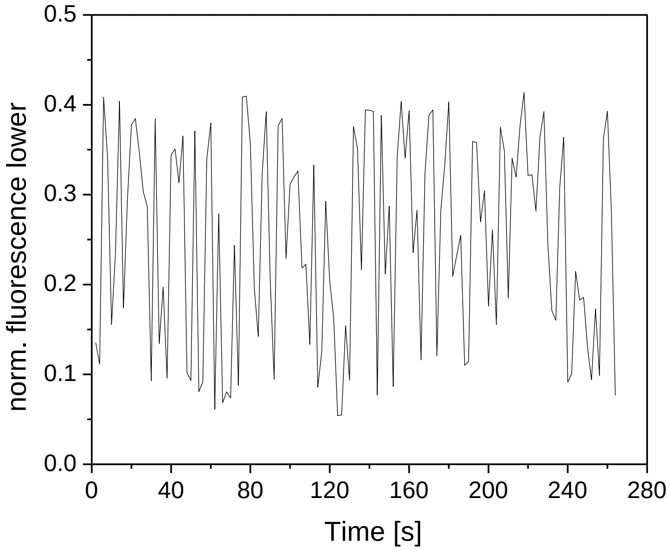

$$\Phi_1 - \Phi_2 = 16\omega_{rec}T$$

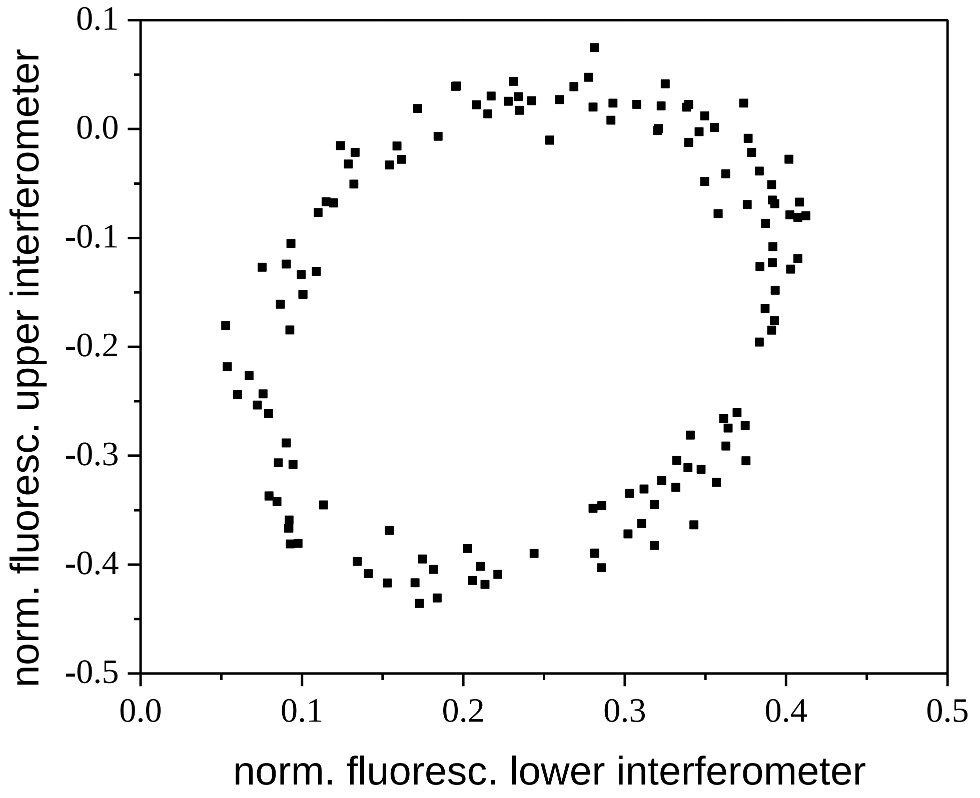
Also cancels gravity

# Results

6th order Bragg diffraction, T =1 ms

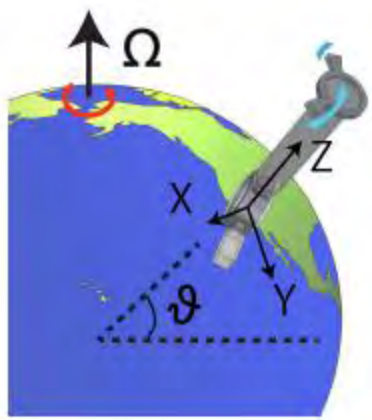


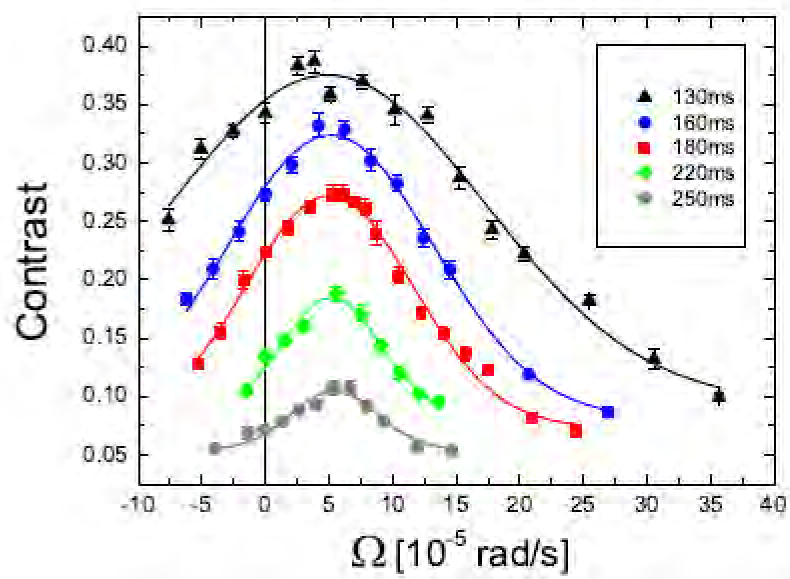



# Coriolis force

$$\vec{\delta} = 4nv_r\Omega_{\oplus}T(T+T')\cos\vartheta(1,0,0).$$



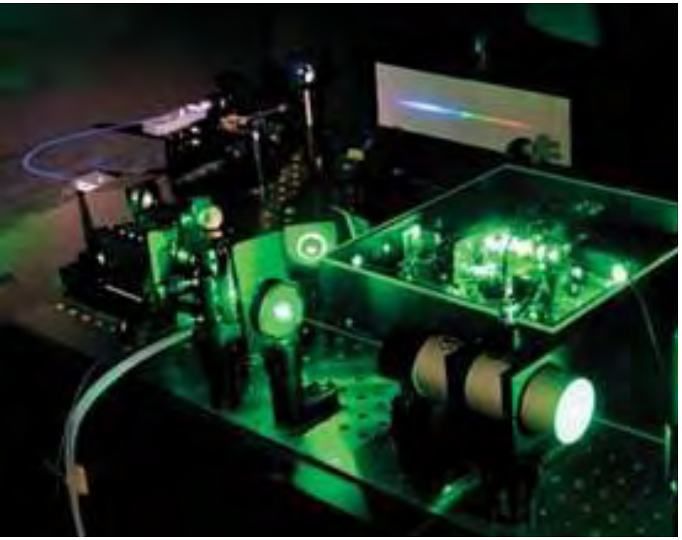



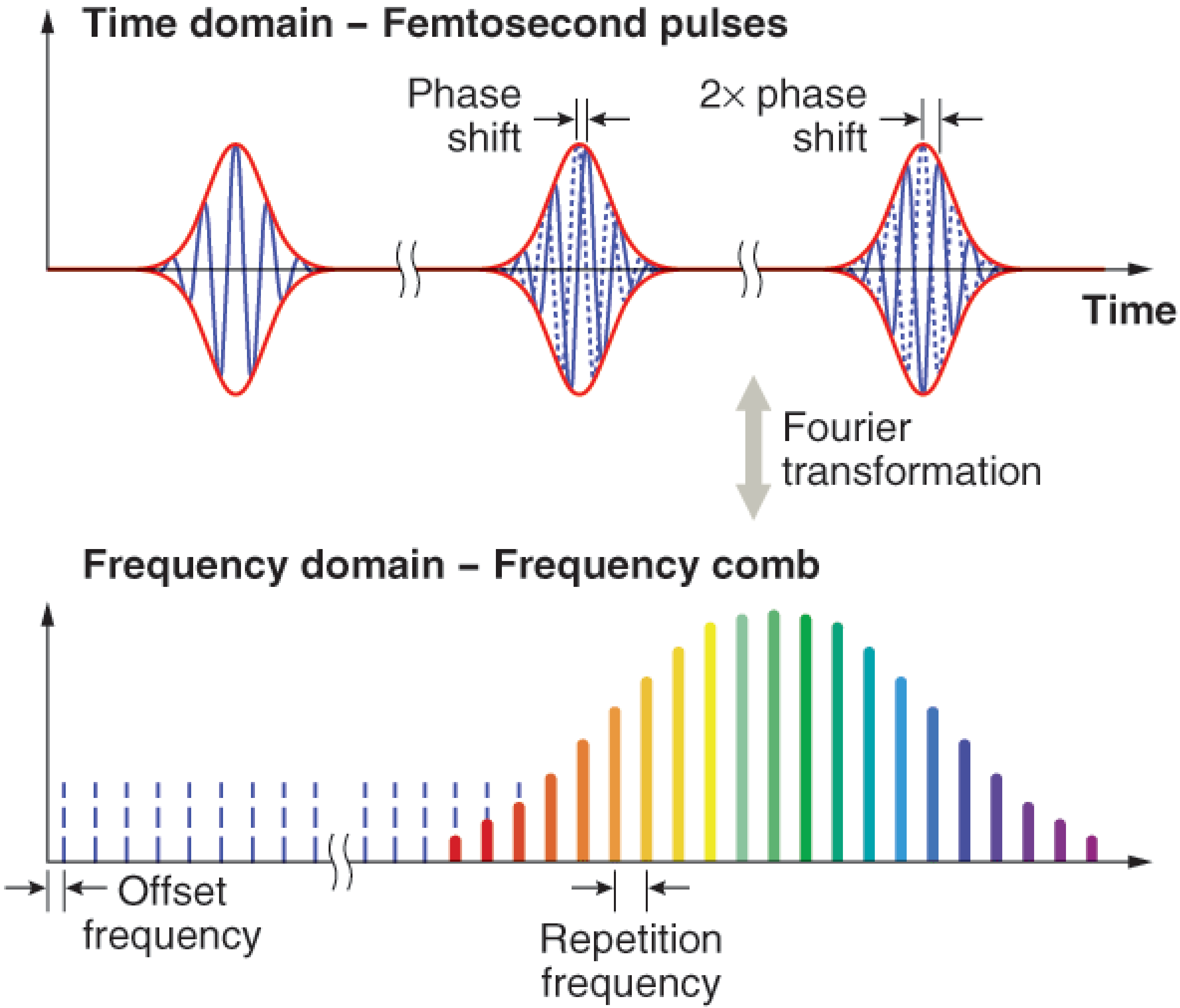

- Rotation causes interferometer not to close
- Can be cancelled by tip tilt-mirror
- Improved contrasy (350%), T, and sensitivity
- World's most sensitive atom interferometer  (10 ħk, 250 ms)

# What does this have to do with a Compton clock?

$$\omega = \frac{mc^2}{\hbar}\frac{1}{n^2}$$

# Frequency Comb





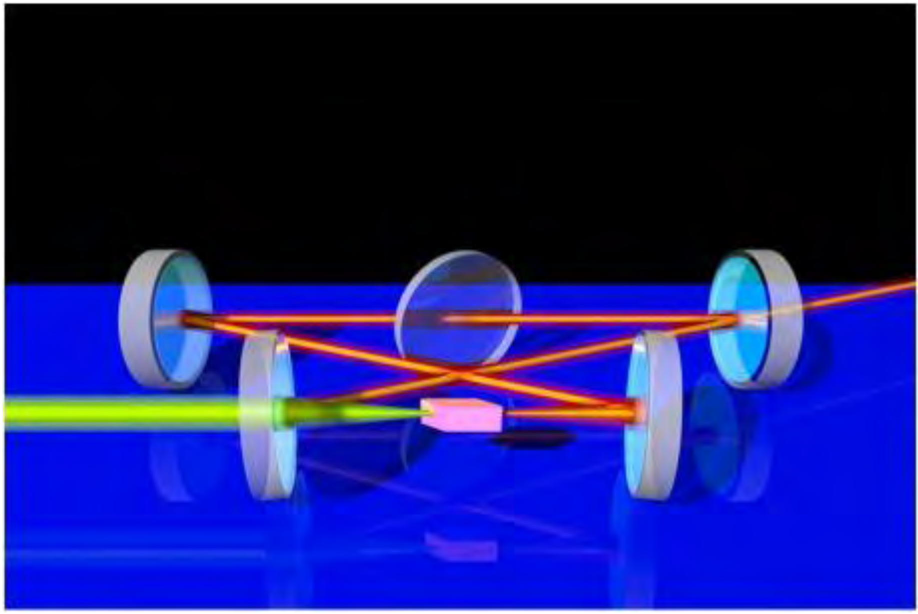

$$f_n = nf_r + f_{offset}$$


[1] Theodor Hänsch, Nobel Lecture, http://nobelprize.org/nobel_prizes/physics/laureates/2005/hansch-lecture.html

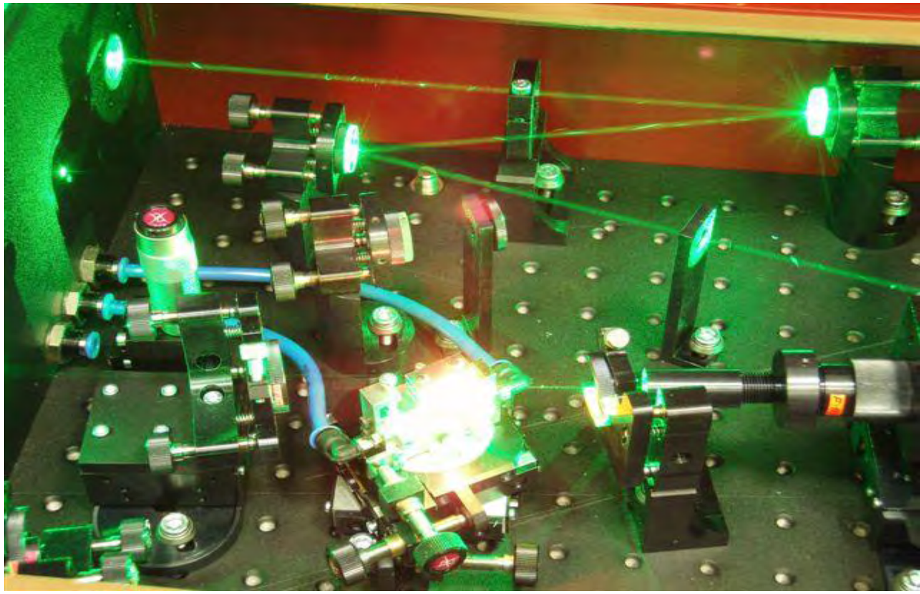
[2] J. Hall, Nobel Lecture, http://nobelprize.org/nobel_prizes/physics/laureates/2005/hall-lecture.html

# Ti:Sapphire systems

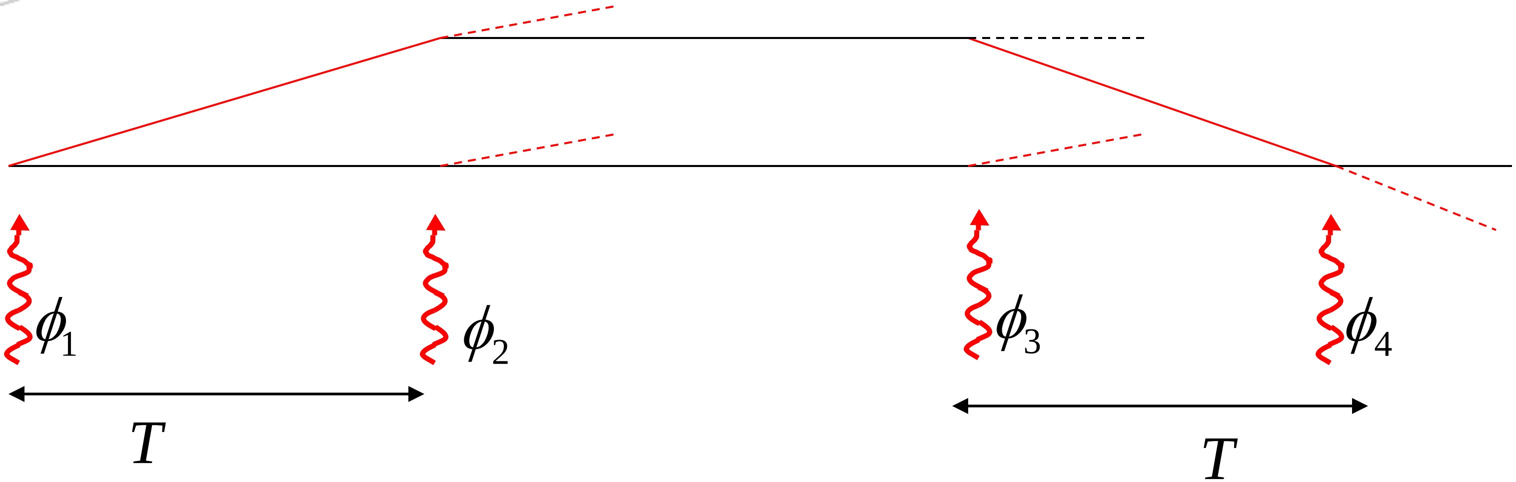
• 500-1000 nm,
• shorter wavelengths by frequency doubling
• 100s of MHz repetition rate



NIST



# Ramsey-Borde interferometer



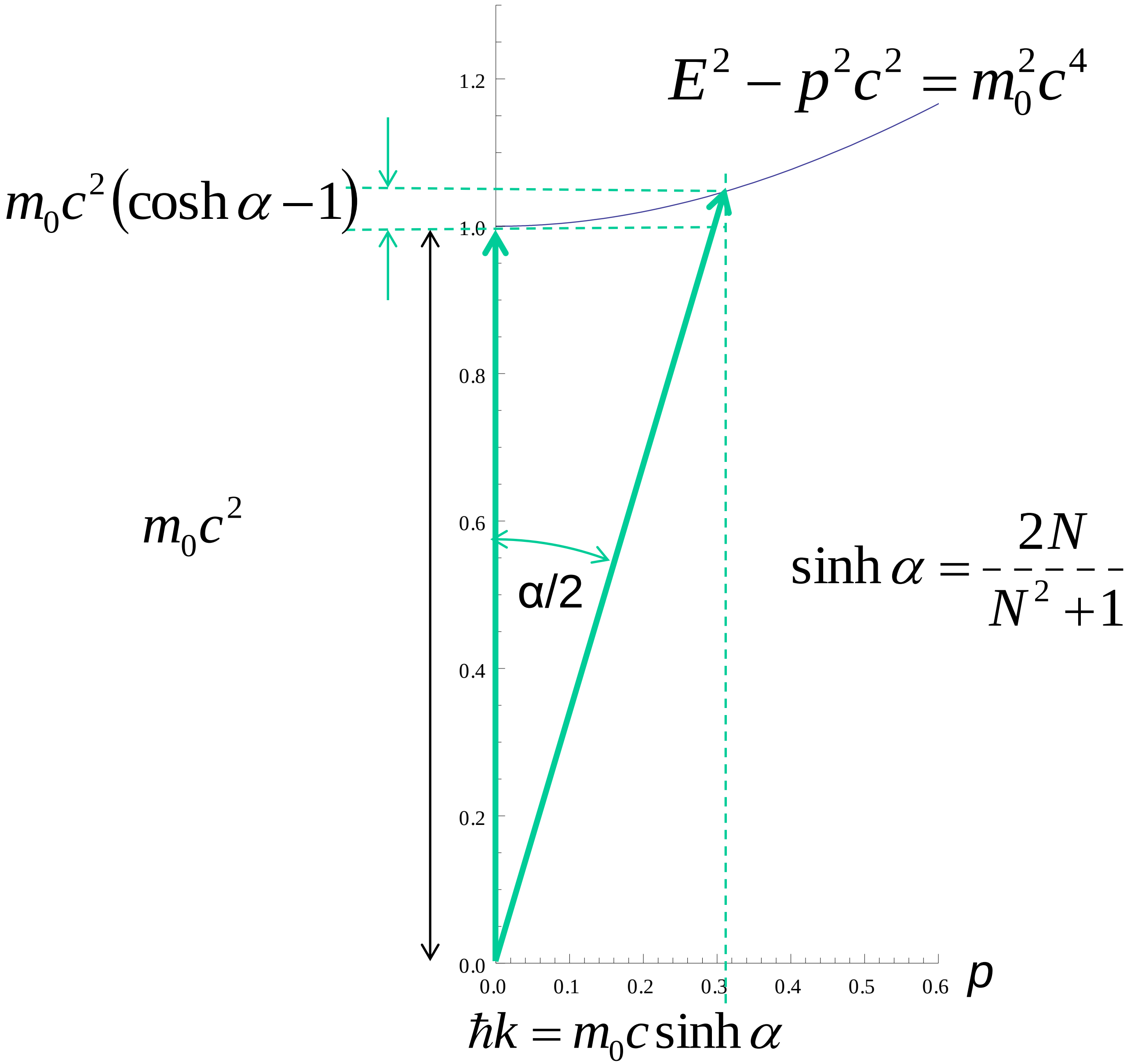

$$\Phi = 8n^2 \frac{\hbar k_L^2}{2m} T = 4n^2 \frac{\omega_L^2}{\omega_C} T$$

Use frequency comb to stabilize $k = \omega_L / c = N\omega_r$

Then find $\Phi_1 = 4n^2 N\omega_C T$

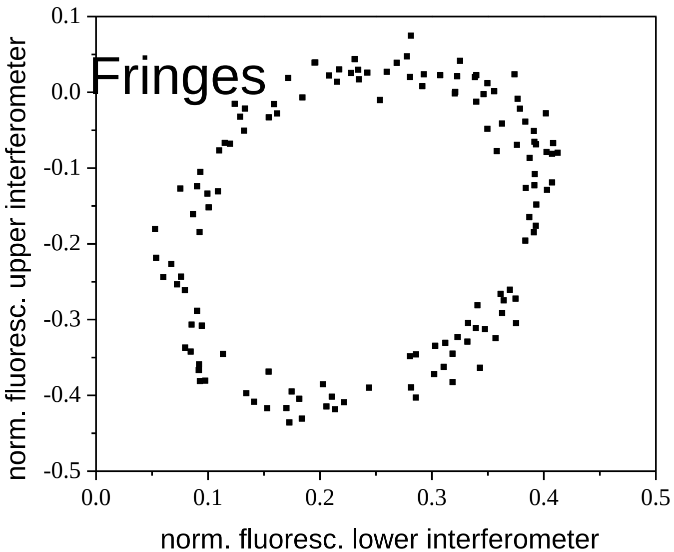
$$\omega_C : \omega_L : \omega_r = N^2 : N : 1$$

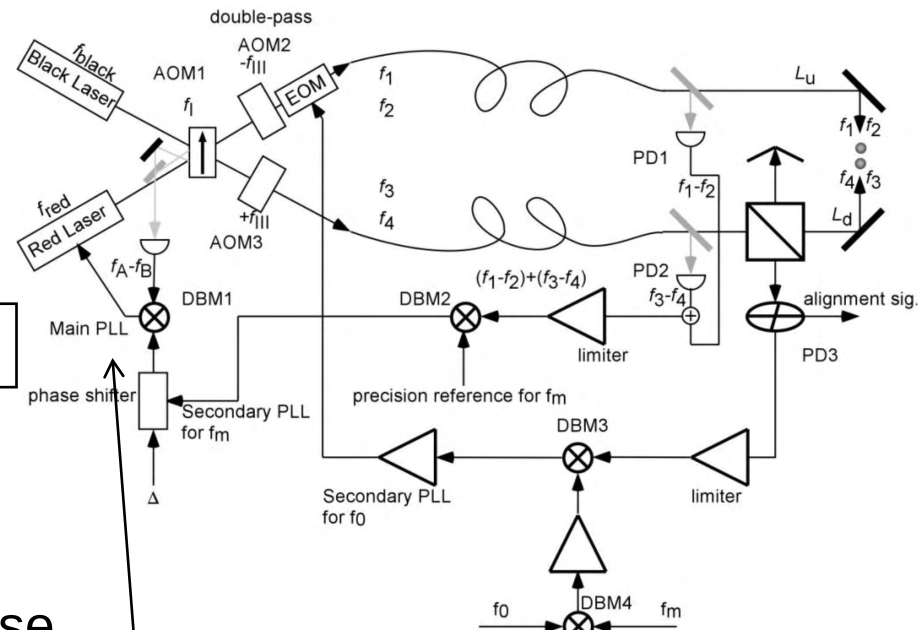
# Relativistic derivation



# Compton clock

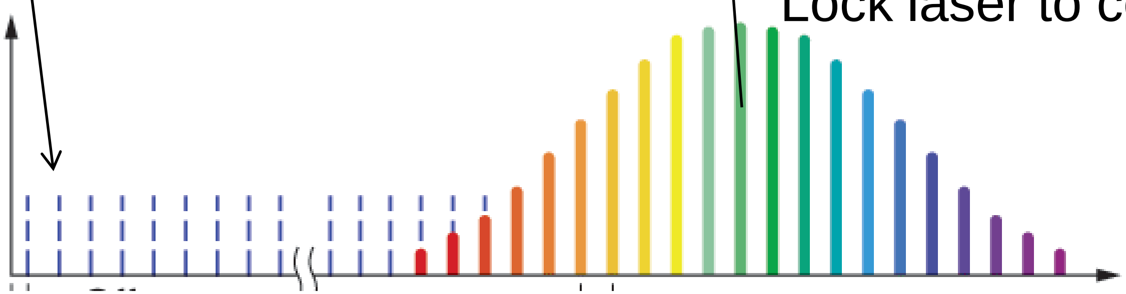
Atom interferometer




Lock comb to zero phase

Lock laser to comb line

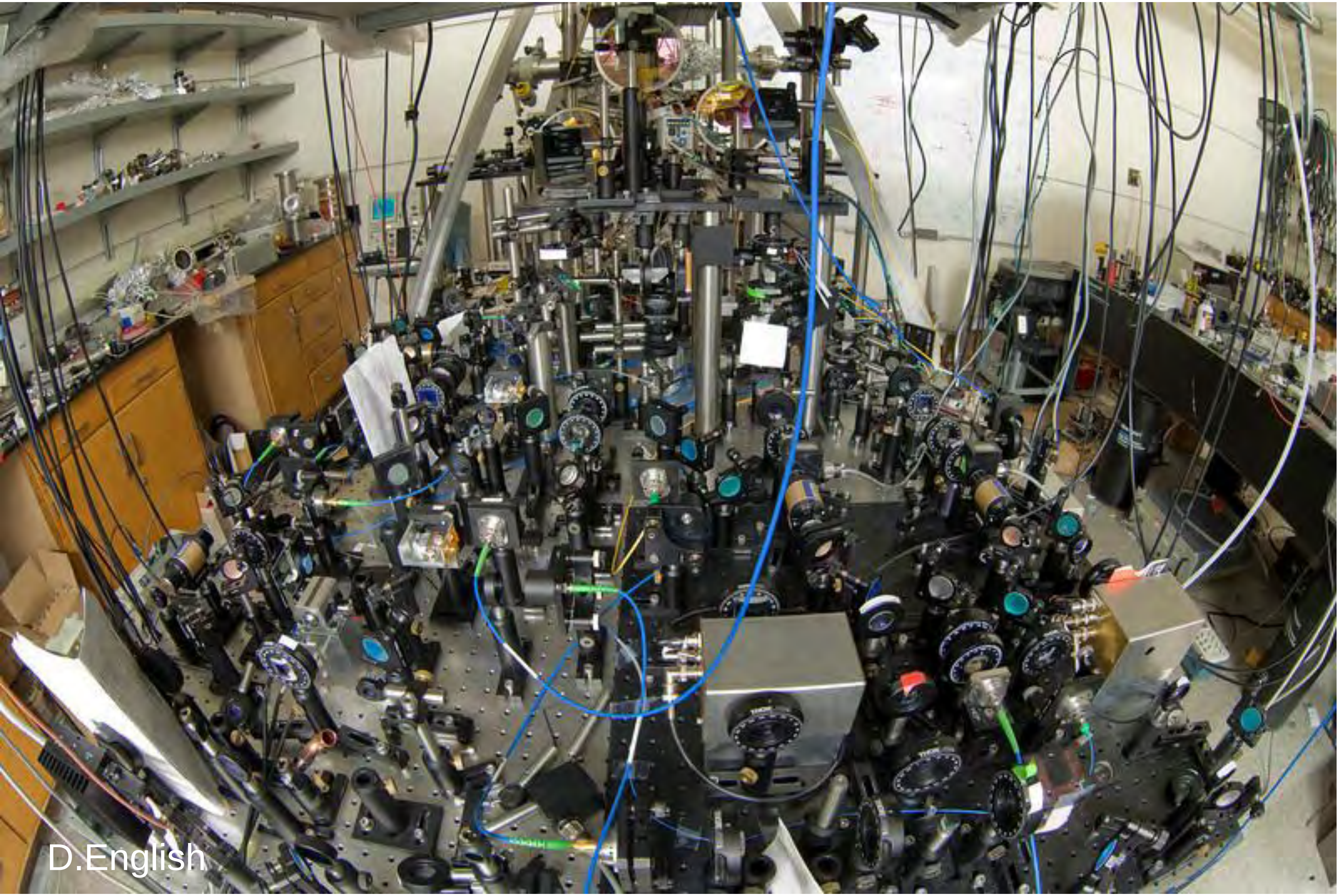


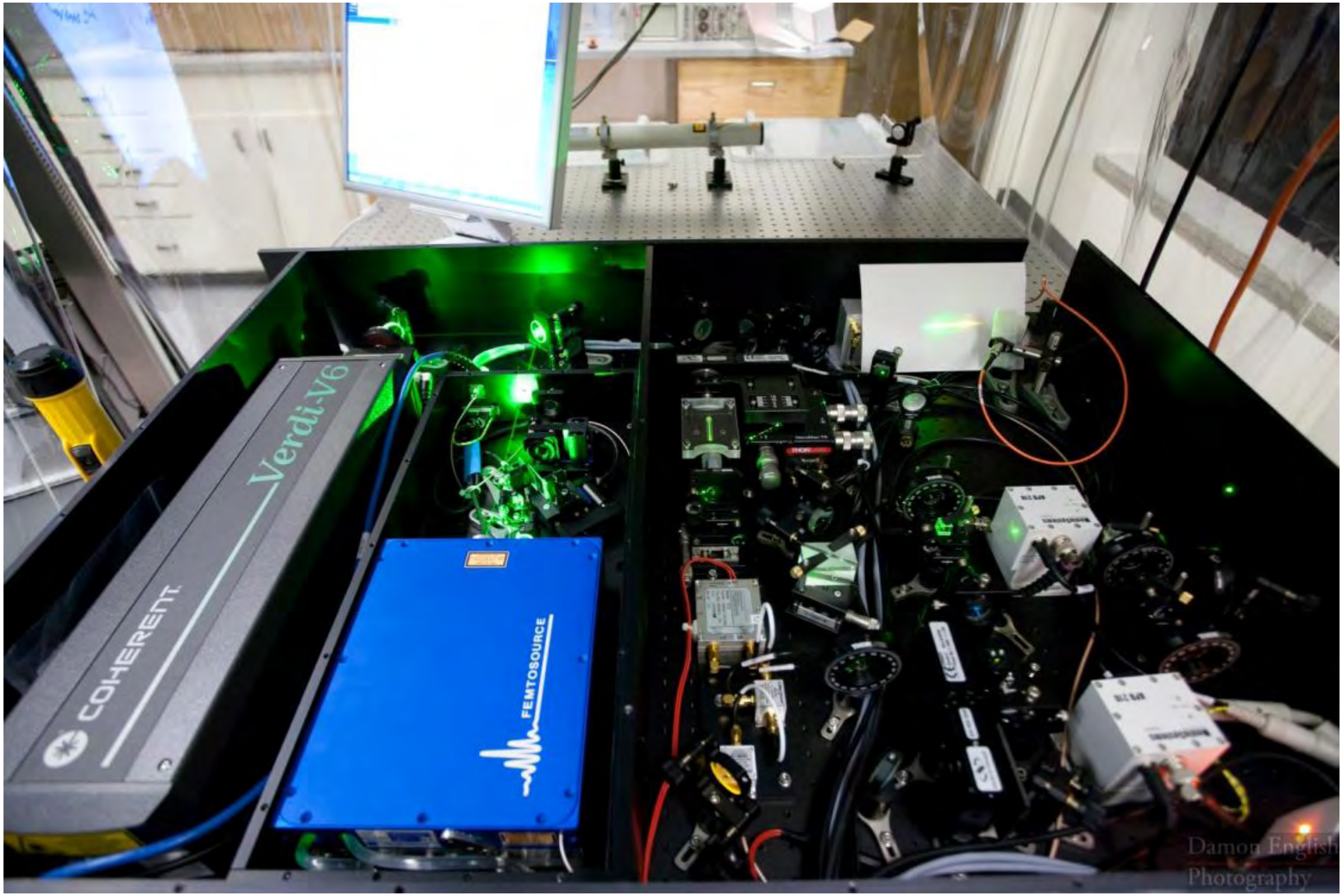
$$\omega_C : \omega_L : \omega_r = N^2 : N : 1$$

3x10[25] Hz

3x10[14] Hz

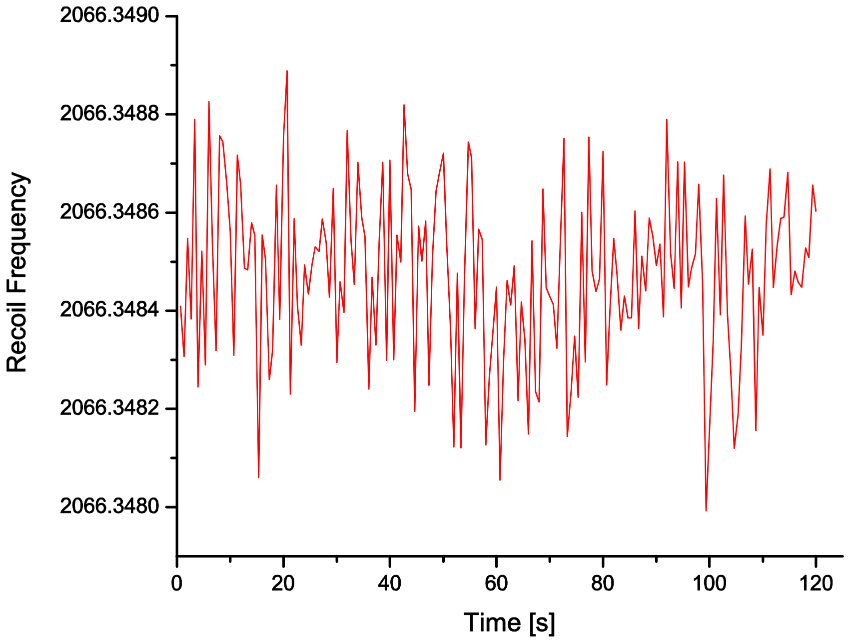
2 kHz

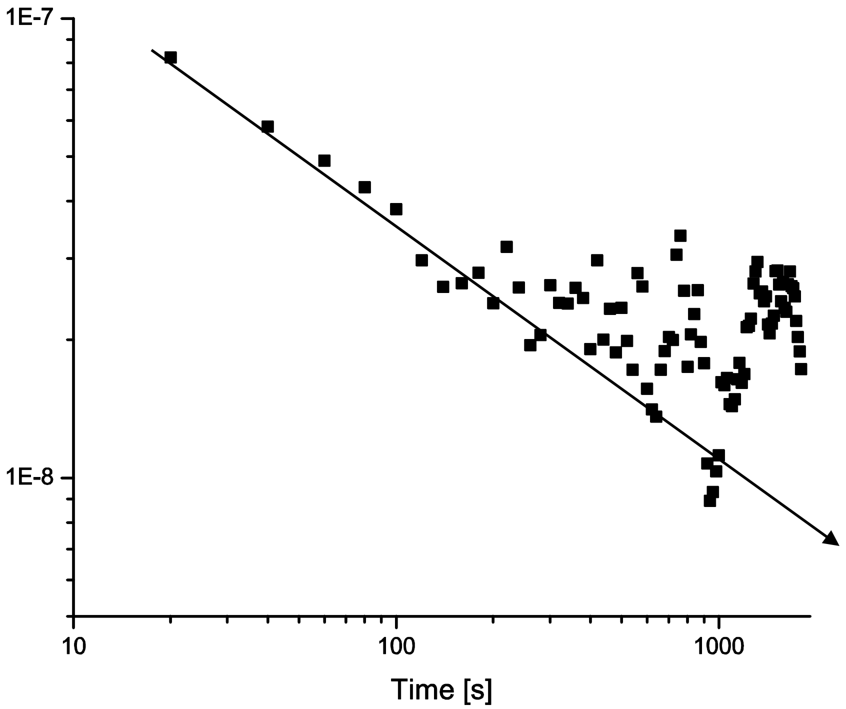

D.English



COHERENT
Verdi-V6
FEMTOSOURCE
Damon English
Photography

# Preliminary data



# Conclusion

- A rock redshifts, time dilates, will soon time the length of the day

- Gravitational AB effect: the phase shift is from the potential, not the force

- Compton clock: divide $3\text{x}10^{25}$ Hz using relativity with self-referenced atom velocity

- $10^{-9}$ redshift test

- Large momentum transfer, conjugate interferometers

Time is an illusion, lunchtime doubly so

*Douglas Adams*

$$\Psi = e^{i\frac{mc^2}{\hbar}t}\psi$$

A rock is a clock

# Schrodinger equation from GR

$$S = \int mc^2 \sqrt{1 + h_{\mu\nu} u^\mu u^\nu}\, d\lambda$$

$$\approx \int mc^2 \left( 1 + \frac{1}{2} h_{00} + h_{0j} \frac{v^j}{c} - \frac{1}{2} \left( \delta_{jk} - h_{jk} \right) \frac{v^j}{c} \frac{v^k}{c} \right) dt$$

$$\Psi(t+\varepsilon, x_A) = N \int d^3\xi \Psi(t, x_A - \xi) \exp\left( i \frac{mc^2}{\hbar} \left( 1 + \frac{1}{2} h_{00} \right) \varepsilon \right) \exp\left( -\frac{1}{2} A_{jk} \xi^j \xi^k + B_j \xi^j \right)$$

$$\int e^{-\frac{1}{2} A_{jk} \xi^j \xi^k + B_j \xi^j} d^3\xi = \frac{(2\pi)^{3/2}}{\sqrt{\det A}} e^{-\frac{1}{2} B_j (A^{-1})_{jk} B_k}$$

$$i\hbar \frac{d}{dt} \Psi = -mc^2 \frac{1}{2} h_{00} \Psi - \frac{\hbar^2}{2m} \left( \vec{\nabla} - m\vec{h} \right) \Psi$$